VC 12309 135

# CBM003 ADD/CHANGE FORM

APPROVED APR 24 2013

| ☒ Undergraduate Council | or | ☐ Graduate/Professional Studies Council |
|---|---|---|
| ☐ New Course ☒ Course Change | | ☐ New Course ☐ Course Change |
| Core Category: ____ Effective Fall 2015 | | Effective Fall |

COMPONENT AREA OPTION (a) 090

1. Department: Mathematics College: NSM
2. Faculty Contact Person: Charles Peters Telephone: 743-3516 Email: charles@math.uh.edu
3. Course Information on New/Revised course:
   - Instructional Area / Course Number / Long Course Title: MATH / 1431 / Calculus I

RECEIVED APR - 4 2013

   - Instructional Area / Course Number / Short Course Title (30 characters max.) MATH / 1431 / Calculus I
   - SCH: 4.00 Level: FR CIP Code: 27.0101.00.01 Lect Hrs: 4 Lab Hrs: 2
4. Justification for adding/changing course: **To meet core curriculum requirements**
5. Was the proposed/revised course previously offered as a special topics course? ☐ Yes ☒ No
   If Yes, please complete:
   - Instructional Area / Course Number / Long Course Title: ____ / ____ / _____
   - Course ID: _____ Effective Date (currently active row): _____
6. Authorized Degree Program(s): _____
   - Does this course affect major/minor requirements in the College/Department? ☐ Yes ☒ No
   - Does this course affect major/minor requirements in other Colleges/Departments? ☐ Yes ☒ No
   - Can the course be repeated for credit? ☐ Yes ☒ No (if yes, include in course description)
7. Grade Option: Letter (A, B, C ...) Instruction Type: lecture ONLY (Note: Lect/Lab info. must match item 3, above.)
8. If this form involves a change to an existing course, please obtain the following information from the course inventory: Instructional Area / Course Number / Long Course Title
   MATH / 1431 / Calculus I
   - Course ID: 31104 Effective Date (currently active row): 8272012
9. Proposed Catalog Description: (If there are no prerequisites, type in "none".)
   Cr: 4. (4-0). Prerequisites: MATH 1330. Description (30 words max.): Calculus of rational functions. limits, derivatives, applications of the derivative, antiderivatives, the definite integral with applications, mean value theorem, fundamental theorem of calculus, and numerical integration.
10. Dean's Signature: _____ Date:_____

    Print/Type Name: _____

# REQUEST FOR COURSES IN THE CORE CURRICULUM

Originating Department or College: Department of Mathematics
Person Making Request: Charles Peters Telephone: 713-743-3516
Email: charles@math.uh.edu
Dean's Signature: _____ Date: 2/13/2013

Course Number and Title: MATH 1431: Calculus I
Please attach in separate documents:

☒ Completed CBM003 Add/Change Form with Catalog Description

☒ Syllabus

List the student learning outcomes for the course (Statements of what students will know and be able to do as a result of taking this course. See appended hints for constructing these statements):
Students will understand and be able to apply the ideas of differential and integral calculus to problems involving instantaneous rates of change, properties of curves, areas bounded by curves, motions of accelerated bodies, volumes, and work. They will develop proficiency in the rules and techniques of single-variable calculus, including derivatives of various combinations of functions, the chain rule, substitution, the mean value theorems, and the fundamental theorem of calculus. Students will be able to use graphical information and symbolic expression simultaneously in solving mathematical problems. They will be able to translate ordinary language descriptions of problems into mathematical expression, derive solutions by rigorous mathematical methods, interpret their results, and explain them.

Component Area for which the course is being proposed (check one):
***Note:** If you check the Component Area Option, you would need to also check a Foundational Component Area.

☐ Communication
☒ Mathematics
☐ Language, Philosophy, & Culture
☐ Creative Arts
☐ Life & Physical Sciences
☐ American History
☐ Government/Political Science
☐ Social & Behavioral Science
☐ Component Area Option

Competency areas addressed by the course (refer to appended chart for competencies that are required and optional in each component area):

| | |
|---|---|
| ☒ Critical Thinking | ☐ Teamwork |
| ☒ Communication Skills | ☐ Social Responsibility |
| ☒ Empirical & Quantitative Skills | ☐ Personal Responsibility |

Because we will be assessing student learning outcomes across multiple core courses, assessments assigned in your course must include assessments of the core competencies. For each competency checked above, indicated the specific course assignment(s) which, when completed by students, will provide evidence of the competency. Provide detailed information, such as copies of the paper or project assignment, copies of individual test items, etc. A single assignment may be used to provide data for multiple competencies.

Critical Thinking:
Several examples of exercises and assignments addressing critical thinking competencies are attached.

Communication Skills:
See attached.

Empirical & Quantitative Skills:
See attached.

Teamwork:
Click here to enter text.

Social Responsibility:
Click here to enter text.

Personal Responsibility:
Click here to enter text.

Will the syllabus vary across multiple section of the course? ☐ Yes ☒ No

If yes, list the assignments that will be constant across sections:
Click here to enter text.

Inclusion in the core is contingent upon the course being offered and taught at least once every other academic year. Courses will be reviewed for renewal every 5 years.

The department understands that instructors will be expected to provide student work and to participate in university-wide assessments of student work. This could include, but may not be limited to, designing instruments such as rubrics, and scoring work by students in this or other courses. In addition, instructors of core courses may be asked to include brief assessment activities in their course.

Dept. Signature: Charles Potts

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| ☒ Undergraduate Council | or | ☐ Graduate/Professional Studies Council |
|---|---|---|
| ☐ New Course ☒ Course Change | | ☐ New Course ☐ Course Change |
| Core Category: Math/Reason Effective Fall ~~2013~~ 2014 | | Effective Fall 2013 |

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Email: charles@math.uh.edu
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Dept. Signature: _____

The following courses have been reviewed and approved by the NSM Curriculum Committee to meet the new core requirements. Given the length of the individual submissions I have elected to submit these requests by electronic means only.

**Natural Sciences: Core Courses**

BIOL 1309 – Human Genetics and Society

BIOL 1310 – General Biology

BIOL 1320 – General Biology

BIOL 1361 - Introduction to Biological Science I

BIOL 1362 - Introduction to Biological Science II

CHEM 1301 – Foundations of Chemistry

CHEM 1331 – Fundamentals of Chemistry I

CHEM 1332 – Fundamentals of Chemistry II

GEOL 1302 - Introduction to Global Climate Change

GEOL 1330 - Physical Geology

GEOL 1340 - Introduction to Earth Systems

GEOL 1350 - Introduction to Meteorology

GEOL 1360 - Introduction to Oceanography

GEOL 1376 - Historical Geology

PHYS 1301 - Introductory General Physics I

PHYS 1302 - Introductory General Physics II

PHYS 1321 - University Physics I

PHYS 1322 - University Physics II

**Mathematics: Core Courses**

MATH 1310 – College Algebra

MATH 1311 – Elementary Mathematical Modeling

**Math/Reasoning: Core Courses**

COSC 1306 – Computer Science and Programming

MATH 1330 - Precalculus

MATH 1431 - Calculus I

MATH 1432 – Calculus II

MATH 2311 – Introduction to Probability and Statistics

**Writing in the Disciplines: Core Courses**

BCHS Biochemistry Lab II

BIOL 3311 – Genetics Lab

PHYS 3313 – Advanced Lab I

Ian Evans

Associate Dean

4/4/13

# MATH 1431

## Section 13194 MWF 9:00 - 10:00 SEC 100
## Section 13201 MWF 12:00 - 1:00 SR 117

**Instructor:** Pam Balthazar, 620 PGH, pamb@math.uh.edu

**Office Hours (held in CASA):** **Monday 1:00 p.m. - 3:00 p.m.**

**Wednesday 1:00 p.m. - 5:00 p.m.**

**Course Learning Materials:** The textbook, online quizzes, EMCF assignments, Discussion Board and additional help materials will be made available by logging into *CourseWare* at http://www.casa.uh.edu. The first portion of these materials is freely available for the first two weeks of class. All students must purchase a ***Course Access Code*** and enter it on *CourseWare* by the first day of the third week of class to continue accessing the course learning materials. A ***Course Access Code*** can be purchased for about $50 from the University Bookstore. If you want a physical copy of the text for the course, then purchase *CALCULUS, 9th edition.* Authors: Salas/Hille/Etgen. Publisher: John Wiley & Sons, Inc. **Note: You do not need to purchase a physical copy of this text.** You will have access to the text electronically on *CourseWare* once you enter your ***Course Access Code***. Also, even if you purchase a physical copy of the text, you will still need the ***Course Access Code*** to access the additional learning materials, including the online electronic quizzes and EMCF assignments.

**Homework:** Homework will usually be collected on Mondays in recitation, starting the third week of class, except on weeks when recitation does not meet. In these cases, additional information will be given. A list of problems will be posted on the course homepage in a pdf file for completing the homework. You must print off this file and provide your work and answers on the sheets. You may print double sided to save paper. Staple them in the upper left hand corner, and make sure that all information (name, etc.) is provided. If you do not follow the given directions, you will receive a ZERO. You must attempt ALL of the problems in order to receive ANY credit. Your work on your homework will not be returned. Other suggested homework problems can be found on the syllabus.

**Daily Grades:** Daily grades will be given in lecture beginning the first day of the third week of class. You need to purchase a course packet of Course/Section specific Popper Forms for Math 1431 from the bookstore. You must bring one of these forms to class every day beginning week 3. No other form will be accepted. Questions will be asked in lecture at random times. You will mark your answers on your bubble form and turn it in when requested. Your forms will not be returned. FYI: Your grading ID is your PeopleSoft ID.

**EMCF:** "EMCF" stands for "Electronic Multiple Choice Form". EMCF assignments are answered on *CourseWare* using the EMCF tab. The questions will be posted on the EMCF assignment page. EMCF assignments will generally be given three times a week. Check the EMCF assignment page and/or the homepage

for more details.

**Weekly Written Quizzes:** Quizzes will be given every week on Friday in lab. Quizzes will be returned in lab. There may be quizzes on Mondays.

**Weekly Online Quizzes:** Online quizzes will be given most weeks. You may attempt these quizzes as many as 20 times. The highest grade will be used for your score. If you fail to reach 70% during three weeks of the semester, **I have the option** to drop you from the course. You can access the quizzes by logging into CourseWare at http://www.casa.uh.edu/. More information can be found here. The quiz calendar is available from the course homepage. Quizzes **will not** reopen once they have closed.

**Exams:** All sections of Math 1431 take common exams. Four exams are given during the semester. The first exam is online and is available the first day of class, and the others will be taken in CASA (located on the second floor of Garrison). You can access the scheduler for these exams by logging into CourseWare at http://www.casa.uh.edu/. Each exam, after the first, will consist of both multiple choice and written questions. The multiple choice questions will be machine graded. The written questions will be graded by the instructors and teaching assistants for all sections of Math 1431, and they will be returned in lab. The scheduler will be available 2 weeks prior to the start of the exam cycle. More information can be found here.

**Final Exam:** A comprehensive final exam will be given in CASA. You can access the scheduler for this exam by logging into CourseWare at http://www.casa.uh.edu/. More information can be found here.

## Grades:

400 points determined by exams 1, 2, 3 and 4
150 points determined by online quizzes and homework (equally weighted)
100 points determined by daily grades (poppers and EMCFs) and written quizzes (equally weighted)
200 points determined by the final exam
850 points total

**Note:** The percentage grade on the final exam can be used to replace your lowest test score.

90% and above - A
at least 80% and below 90%- B
at least 70% and below 80% - C
at least 60% and below 70% - D
below 60% - F

**Attendance is Mandatory!!** Attendance will be taken in lab, and the poppers will be used to determine your attendance in lecture. I will allow you a total of 3 unexcused absences from lecture and lab (total). You may lose 1% of your grade for every unexcused absence from lecture or lab after the third. Documented University of Houston excused absences will be permitted.

**Whenever possible, and in accordance with 504/ADA guidelines, we will attempt to provide reasonable academic accommodations to students who request and require them.**

# Math 1431 Calculus 1 – Topics List

## LIMITS AND CONTINUITY

Definition of Limit
Continuity

## DIFFERENTIATION

Differentiation Formulas
Derivative as a Rate of Change
Chain Rule
Differentiating the Trigonometric Functions
Implicit Differentiation
Rates of Change per unit time
Differentials

## THE MEAN-VALUE THEOREM

Newton-Raphson Approximations
Mean-Value Theorem
Increasing/Decreasing Functions
Extrema
Optimization
Concavity and Points of Inflection
Curve Sketching

## INTEGRATION

Fundamental Theorem of Integral Calculus
Area problems
Indefinite Integrals
U-Substitution; Change of Variables
Average Value

## SOME APPLICATIONS OF THE INTEGRAL

More on Area
Volume

Below are some representative questions from various assessment pieces that demonstrate our commitment to these objectives. All of the following examples are from exams given during the semester.

## Part A: Critical Thinking Skills

**Multiple Choice:**

Evaluate the following limit: $\lim_{x \to 0} \frac{x \cos(6x)}{\tan(7x)}$ .

This question brings to the fore a student's grasp of the basic information about the limit as $x$ goes to zero and getting an indeterminate answer. The student must analyze the result and then utilize what they learned from trigonometric limits and identities to get their answer.

**Multiple Choice:**

The functions $f$ and $g$ are differentiable and $h(x) = 3(f(x)+4)^2 + 4g(x)$. Use the information below to find $h'(4)$:

$$f(4) = -5, f'(4) = 6$$
$$f(2) = -3, f'(2) = 2$$
$$g(4) = 2, g'(4) = -2$$
$$g(-3) = 3, g'(-3) = -6$$

The question gives all the information needed. It is up to the student to recognize the composite function and then to find its derivative. They must then evaluate the derivative at the given value. This really tests whether or not a student understands the derivative value using the chain rule.

**Free Response:**

Given: $x^2 + 2xy + 3y^2 = 22$

**Part a:** Give a formula for $dy/dx$ in terms of $x$ and $y$.
**Part b:** Give the equation for the normal line to the graph at the point (-1, 3).

With this question the student must evaluate the derivative, taken implicitly, at a point and then use that answer to find the equation of the line. They must demonstrate an understanding of the derivative as the slope of the tangent line to the graph of the function and then use what they know about slopes of parallel/normal lines in order to provide the equation of the normal line.

**Multiple Choice:**

Given that $f(x) = \frac{1}{6}x^3 - x^2 - 4x - 5$ determine the value(s) of $x$ for which the graph of $f$ has a point of inflection.

The student must compute a derivative of higher order. In this particular problem, they must use the second derivative by recognizing the relationship between the concavity of $f$ and the sign of the second derivative to identify points of inflection as places where concavity changes.

**Free Response:**

Given: $y = x^2 + 3$
$y = x + 5$
**Part a:** Graph the equations on the same axis and shade the region between them.
**Part b:** Give the formula for the area of the region bounded by the curves.

The student must remember graphing from previous courses to graph the given functions. They must then analyze the drawn graph to determine how to find the area of the region by applying the Fundamental Theorem of Calculus to represent the given area.

**Multiple Choice:**

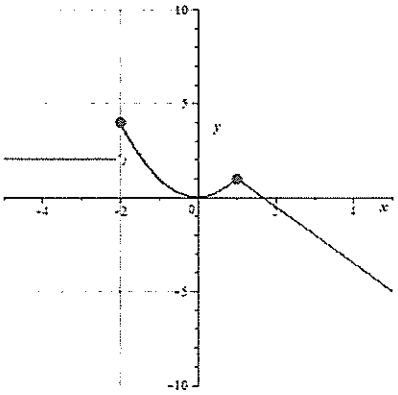
Compute

$$\int_0^1 (12x-8)\left(3x^2-4x\right)^6 dx =$$

The student will use integration techniques, including change of limits for definite integrals, in order to evaluate the given integral. If the student analyzes this problem carefully, they would realize the pattern and be able to solve it quickly. Other students will be more methodical and use u-substitution and change of limits.

**Part B: Communication Skills**

**Free Response:**




Use the graph to answer the following:

Part a: Find *i)* $\lim_{x \to -2^+} f(x)$

*ii)* $\lim_{x \to -2^-} f(x)$

*iii)* $\lim_{x \to 1} f(x)$

**Part b:** Is $f(x)$ continuous at $x = -2$? If not, classify the discontinuity as either a jump discontinuity, a removable discontinuity or an infinite discontinuity. Explain you answer.

**Part c:** Is $f(x)$ continuous at $x = 1$? If not, classify the discontinuity as either a jump discontinuity, a removable discontinuity or an infinite discontinuity. Explain you answer.

The student must understand the concepts of limits and continuity in terms of limits. They must then apply that understanding to determine the limits and continuity at the given values, using their geometric understanding and interpretation of limits/continuity. The student must then clearly express the result, justifying their answer.

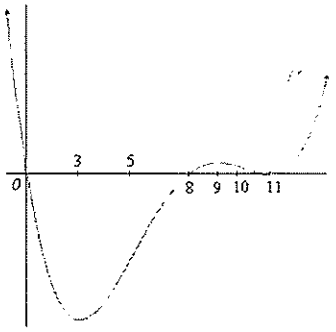
**Free Response:**

Sand is falling off a conveyor onto a conical pile at a rate of 20 cubic feet per minute. The diameter of the base of the cone is twice the altitude. At what rate is the height of the pile changing when it is 8 feet high?

The student must translate the verbal question of what is being asked into mathematical terms and then solve the problem. They must demonstrate an understanding of the interpretation of the derivative as an instantaneous rate of change. They will then create the model for the situation and use it to solve the problem.

**Free Response:**

The graph of the **DERIVATIVE of** $f$ is shown below.




*i.* Find and classify the critical values of $f$.
*ii.* Give the interval(s) where $f$ is decreasing.
*iii.* Give the interval(s) where $f$ is concave up.
*iv.* Give the values(s) of $x$ where $f$ has inflection.

In this problem, the student must communicate their understanding of a derivative given in a graphical context. They must analyze the graph, translate that information into numerical data they can work with, and then define the relationships between the data and the behavior of the function. They must then clearly answer the questions.

**Free Response:**

A function is given below along with both its first and second derivatives. List the domain, vertical and horizontal asymptotes, critical numbers, intervals of increase, intervals of decrease, inflection points, intervals of concave up, and intervals of concave down for the function. Then graph the function and carefully label the $y$-intercept, local extrema, and point(s) of inflection.

$$f(x) = -\frac{x}{(x+1)^2}$$
$$f'(x) = \frac{x-1}{(x+1)^3}$$
$$f''(x) = -\frac{2(x-2)}{(x+1)^4}$$

This particular problem has the student showing their understanding of the key concepts of derivatives and their relationships to each other and to the function. The student must interpret the results and communicate them through numerical and graphical expressions.

**Free Response:**

A manufacturer wants to design an open box with a square base and a surface area of 108 square inches.

(i) Express the volume of this box as a function of a single variable and give the domain of this function.
(ii) Determine the dimensions of the box with maximum volume.

The student must develop a model for this situation by translating a verbal description into equation(s) to be used to optimize the given situation, using derivatives and their knowledge of global and local extrema. They must then communicate the answer to the question being asked, which is not what they get when they solve their equation.

**Multiple Choice:**

Find the slope of the tangent line to the graph of $y = 5\sin(3x)$ at the point where $x = \pi/3$.

This is a very basic knowledge question. However, the student must interpret the question and then express the solution in the correct form.

## Part C: Empirical and Quantitative Skills

**Free Response:**

Find the function $f$ that satisfies these conditions:

$f''(x) = x^2 - 4x, \qquad f'(0) = 5, \qquad f(0) = -2$

The student is given the second derivative and must use the given data to find the function that satisfies the conditions. They must find the specific antiderivative using the given conditions.

**Free Response:**

Given $f(x) = -x^2 + 2$

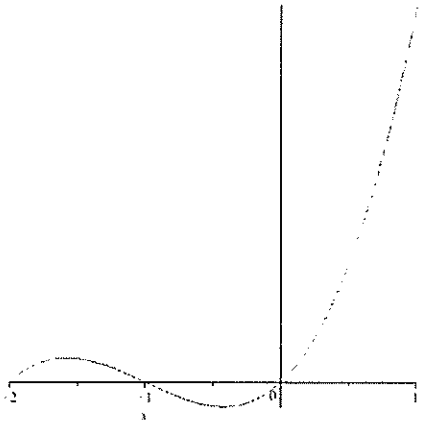
**Part a:** Sketch the graph of the function with rectangles drawn to represent the **lower** sum of $f$ over the interval [-2, 2] with respect to the partition P = {-2,-1, 1, 2} then give an expression to compute the **lower** sum, $L_f(P)$.

**Part b:** Sketch the graph of the function with rectangles drawn to represent the **upper** sum of $f$ over the interval [-2, 2] with respect to the partition P = {-2,-1, 1, 2} then give an expression to compute the **upper** sum, $U_f(P)$.

The student must be able to sketch the graph and then represent the given Riemann sum. They must interpret the graph they drew in order to compute the Riemann sum. They must then be able to evaluate the numerical solutions. When they have finished, they should analyze their answers to verify that they make sense in the context of the problem.

**Multiple Choice:**

The graph of $f$ is shown below on the interval [-2, 1].



The area bounded between the graph of $f$ and the $x$-axis on [-2, -1] is 2, the area bounded between the graph of $f$ and the $x$-axis on [-1, 0] is 2, and the area bounded between the graph of $f$ and the $x$-axis on [0, 1] is 18. Determine $\int_{-2}^{1} f(x)dx =$

This problem verifies that the student recognizes the relationship between the graph of a function and the integral of the function. They will use the given data, in terms of area, to compute the requested integral, thereby showing their understanding of the relationship between area and the definite integral.

**Multiple Choice:**

The function $k$ is an antiderivative of the continuous function $g$, and the function $f$ is an antiderivative of the continuous function $h$. Also,

| | | | |
|---|---|---|---|
| $g(1)=2$ | $g(6)=-4$ | $k(1)=-2$ | $k(6)=-5$ |
| $h(1)=7$ | $h(6)=3$ | $f(1)=9$ | $f(6)=4$ |

Compute $\int_{1}^{6} \left(-g(x)-4h(x)\right)dx$

The student must first translate the verbal question in order to know how to solve the problem. When they make the connection between the integral and the given antiderivatives, they can then use the data to compute the solution.

**Multiple Choice:**

Given that

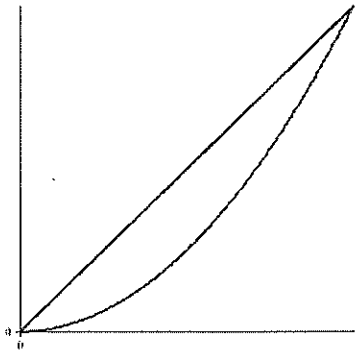
$$\left[ \int_0^1 f(x)\,dx = 2, \qquad \int_0^3 f(x)\,dx = 5, \qquad \int_3^6 f(x)\,dx = 4 \right]$$

Find $\int_6^1 f(x)\,dx$.

The student uses the properties of the definite integral to solve this problem. They must observe and recognize the details in each part in order to find the correct solution.

**Free Response:**

The region bounded by $f(x) = x^2$ and $g(x) = 7x$ is shown below.



**Part a:** Give a formula involving integral(s) for the volume generated when the region is revolved about the $x$-axis.

**Part b:** Give a formula involving integral(s) for the volume generated when the region is revolved about the $y$-axis.

The student must be able to translate the given information into a formula. They must recognize what the question is asking – volume – and use the appropriate integral to model the volume of the sold generated by the given revolution.